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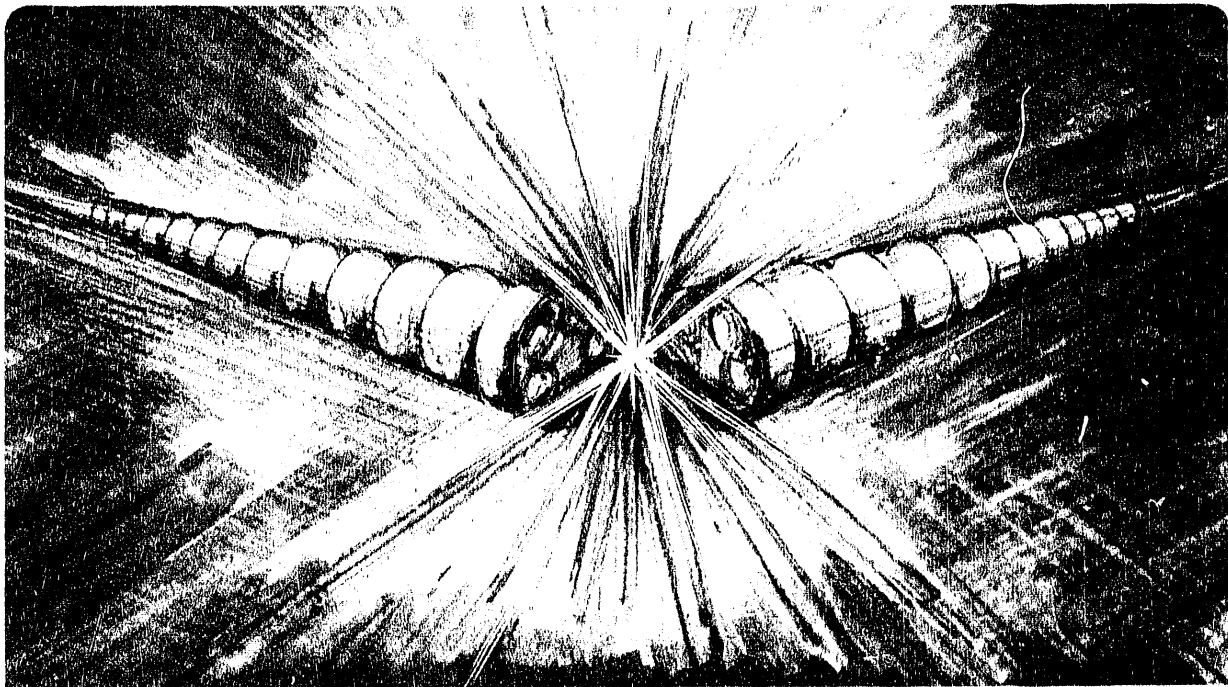## Accelerator & Fusion Research Division

Estimates of SASE Power in the Short Wavelength Region

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Summary

Given a sufficiently bright electron beam, the self-amplified-spontaneous emission (SASE) can provide gigawatts of short wavelength coherent radiation. The advantages of SASE approach are that it requires neither optical cavity nor an imput seed laser. In this note, we estimate the peak power performance of SASE for wavelengths shorter than 1000 Å. At each wavelength, we calculate the saturated power from a uniform parameter undulator and the enhanced power from a tapered undulator.

The method described here is an adaptation of that discussed by L. H. Yu [1], who discussed the harmonic generation scheme with seeded laser, to the case of SASE.

1. **Basic Formula**

- Peak undulator magnetic field of an N_d-F_e-B undulator [2]

$$B_o[T] = 3.44 \exp\left[-5.08 \, (g/\lambda_u) + 1.54 \, (g/\lambda_u)^2\right] \, , \tag{1}$$

where g = magnet gap, λ_u = undulator period.

- Deflection parameter:

$$K = 0.934 \, \lambda_u[cm] \cdot B_o \, [T] \, , \tag{2}$$

- Resonance condition for radiation wavelength λ:

$$\lambda = \frac{\lambda_u \left(1 + K^2/2\right)}{2\gamma^2} \, , \tag{3}$$

where γ = electron energy/rest energy.

- Natural focussing in the undulator for the case of equal horizontal and vertical focussing via parabolic pole shaping:

$$k_\beta = \frac{K\pi}{\gamma \lambda_u} \quad . \tag{4}$$

- Given the rms electron beam emittance ε_x, the rms beam size σ_x is given by

$$\sigma_x^2 = \varepsilon_x/k_\beta \quad , \tag{5}$$

2. Exponential growth

In the linear regime, the electromagnetical field grows exponentially as follows [3]:

$$|A| \sim e^{\mu k_u z} \, , \, k_u = 2\pi/\lambda_u \tag{6}$$

The growth rate μ, taking into account the 3-D and the inhomogeneous spread effects, was first calculated by Yu, Krinsky and Glucstern [4] for the waterbag model and later for general phase space distribution by Chin, Kim and Xie [5]. The result for the Gaussian distribution can be conveniently parameterized in the following form [5]:

$$\log \frac{\mu}{D} = - \left(0.75 + 0.23 \, X + 0.016 \, X^2\right)$$

$$\times \left\{ 1 + \frac{(EB)^2}{(0.17 + 0.0304 \log B)} + \left(41.34 + 3.69 \, X + 3.62 \, X^2\right)\left[S^2 + 2.18 \, S^4 + 70.9 \, S^6\right] \right\}^{1/2} , \tag{7}$$

where

$$D = \left[8 \, \frac{K^2/2}{1 + K^2/2} \, \frac{I}{I_A} \, \frac{1}{\gamma} \right]^{1/2} \cdot [JJ] \quad . \tag{8}$$

Here I = peak current and $I_A = 1.704 \times 10^4$ Amp. Also

$$[JJ] = J_0\left(K^2/4 \left(1 + K^2/2\right)\right) - J_1\left(K^2/4 \left(1 + K^2/2\right)\right) ,$$

and $\qquad E = \varepsilon_x / (\lambda / 4\pi) \qquad$ (scaled emittance)

$$B = k_\beta / (k_u D) \quad , \quad k_u = 2\pi / \lambda_u$$

$$X = \log (E/B)$$

$$S = \sigma_\gamma / (\gamma D) \quad , \quad \sigma_\gamma = \text{rms energy spread.}$$

The formula (7) should not be trusted when $\dfrac{\mu}{D} \le 0.01$.

For a conditioned beam [6], the factor $(EB)^2 / (0.17 + 0.0304 \log B)$ should be replaced by zero.

3. **Saturated power in a uniform parameter undulator**

Let $\Omega(z)$ be the synchrotron oscillation frequency using z as the time. The FEL saturates at $z = L$ where

$$\int_0^L \Omega(z) dz \approx \pi$$

Since $\Omega \propto |A|^{1/2} \propto \exp\left(\dfrac{1}{2} \mu k_u z\right)$, we obtain

$$\Omega(L) \approx \frac{\pi}{2} \mu \, k_u \ .$$

On the otherhand, the FEL efficiency can be expressed as [1]

$$\left\langle \frac{\gamma - \gamma_0}{\gamma_0} \right\rangle = \frac{\Omega^4(L)}{(4\rho)^3 \, k_u^4} \ ,$$

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where ρ is the Pierce parameter related to D by

$$D^2 = \pi \frac{\Sigma}{\lambda \, \lambda_u} (4\rho)^3 \tag{9}$$

Here Σ is the electron beam transverse area (defined by I/Σ = peak current density).

For a Gaussian distribution

$$\Sigma = 2\pi \, \sigma_x^2 \quad . \tag{9}$$

Combining these, the saturated power is estimated to be

$$P_{est} \approx 19.1 \frac{\Sigma}{\lambda \, \lambda_u} D^2 \left(\frac{\mu}{D}\right)^4 P_{beam} \quad , \tag{11}$$

where P_{beam} is the electron beam power.

The noise power at input per unit frequency interval is given by $\rho mc^2 \gamma / 2\pi$ [7]. Also, the bandwidth at saturation is about $\Delta\omega \sim \rho\omega$ [8][9]. The length of the undulator L_{sat} required to read saturation through self-amplified spontaneous emission is therefore, roughly

$$L_{sat} \sim \frac{\lambda_u}{4\pi\mu} \log \left[P_{sat} / \left(\frac{c}{\lambda} \rho^2 \, mc^2 \, \gamma\right) \right] \tag{12}$$

The bucket (half) height at saturation is given by

$$\left(\frac{\delta\gamma}{\gamma}\right)_b = \frac{\Omega(L)}{k_r} = \frac{\pi}{2} \left(\frac{\mu}{D}\right) D \quad . \tag{13}$$

If the length of the undulator L_u is shorter than the saturation length L_{sat}, the power is

$$P_u = P_{sat} \, e^{-2\mu k_u(L_{sat} - L_u)} \; . \tag{14}$$

The corresponding bucket height is

$$\left(\frac{\delta\gamma}{\gamma}\right)_b = \frac{\pi}{2} \left(\frac{\mu}{D}\right) D \cdot e^{-\mu k_u(L_{sat} - L_u)/2} \; . \tag{15}$$

4. Power from tapered undulator

More power can be extracted by employing a tapered the undulator after saturation in the uniform parameter undulator. By solving the KMR equation [10], the power after a quadratic B-field tapering of N_{tape} periods is

$$P_{out} = \frac{Z_o}{4} \left(\frac{e}{mc^2}\right)^2 \frac{K^2/2}{(1 + K^2/2)^2} \frac{1}{\sum} \left\{P_{beam} \, f_{trap} \, [JJ] \, \lambda N_{tape} \sin \psi_r\right\}^2 \tag{16}$$

where $Z_o = 377$ ohms, and f_{trap} is the trapping fraction. Equation (16) is the same as that derived by Yu[1] except for the factor $K^2/2 \, (1 + K^2/2)^2$. In deriving Eq. (16), it is assumed that the efficiency $\eta = P_{out}/P_{beam}$ is small:

$$\eta = P_{out}/P_{beam} \ll 1 \; . \tag{17}$$

Also, since the KMR equation is valid for monochromatic radiation, the radiation bandwidth $\Delta\omega/\omega$ at the beginning of tapered section must satisfy

$$\frac{\Delta\omega}{\omega} \ll \left(\frac{\delta\gamma}{\gamma}\right)_b, \; \eta \; . \tag{18}$$

The trapping fraction f_{trap} may be taken to be

$$f_{trap} = 1, \text{ when } \left(\frac{\delta\gamma}{\gamma}\right)_b > \frac{\sigma_\gamma}{\gamma} \quad ,$$

$$= \left(\frac{\delta\gamma}{\gamma}\right)_b \bigg/ \frac{\sigma_\gamma}{\gamma} \quad , \text{ otherwise } . \tag{19}$$

For the cases we are interested here, the bucket height is usually very large so that $f_{trap} = 1$.
Taking $\sin \Psi_r = 0.5$, Eq. (16) becomes for a Gaussian density distribution

$$P_{out}[W] = 1.44 \; 10^{-11} \frac{K^2/2 \, [JJ]^2}{(1 + K^2/2)^2} \left\{ P_{beam}[W] \frac{\lambda}{\sigma_x} f_{trap} \, N_{tape} \right\}^2 \quad . \tag{20}$$

5.   The Procedure to calculate FEL power at a given λ

Assuming that the total length of the undulator L_{tot} is fixed, the peak power is calculated in the following step starting from a uniform parameter undulator:

i)   Input parameters are $\gamma\varepsilon_x$, I, σ_γ/γ.

ii)   Choose a reasonable value of g.

iii)   Choose λ_u. Determine B_0 (Eq. (1)), K (Eq. (2)), γ (Eq. (3)), k_β (Eq. (4)), ε_x and σ_x (Eq. (5)), and D (Eq. (8)), μ (Eq. (7)), P_{sat} (Eq. (11)), L_{sat} (Eq. (12)).

iv)   If $L_{sat} \le L_{tot}$, then choose $L_u = L_{sat}$. The undulator power P_u is the same as P_{sat} (Eq. (11)). If $L_{sat} > L_{tot}$ then $L_u = L_{tot}$, and P_u is given by Eq. (14).

v)   If $L_{sat} = L_u < L_{tot}$, introduce a tapered section of length $L_{tape} = L_{tot} - L_u = N_{tape} \, \lambda_u$, and calculate the power from Eq. (20).

vi)   Go back to iii) to find λ_u that maximize the output power.

We have calculated the SASE performance in the short wavelength region for the case, $\gamma \varepsilon_x = 1.5 \times 10^{-6}$ m-rad, $I = 1000$ A, $\sigma_\gamma/\gamma = 2.2 \times 10^{-4}$, and $g = 4$ mm, and $L_{tot} = 30$ m. The result is presented in Fig. (1). Without tapering, a peak power of 4.5×10^8, 3.6×10^8, 1.8×10^8, and 6.5×10^5 watts are predicted at wavelength 1000, 100, 20 and 10 Å, respectively. For wavelengths longer than 20 Å, L_{sat} is shorter than 30 m so that a tapered section is introduced to increase the power. At 100 Å, $L_{sat} = 11.6$ m, and the power from a 18.4 m tapered section is 36 GW. At 1000 Å, $L_{sat} = 4.4$ m, and the power calculated from Eq. (20) using $L_{tap} = 25.6$ m is 7.3×10^{11} W. However this is impossible, being larger than the electron beam power (3.25×10^{11} W). By employing a shorter tapered section (5.2 m) the power becomes $P_{tap} = 30$ GW at 1000 Å, corresponding to a 10 % efficiency. Table 1 gives a list of the electron beam and amplifier parameters.

The FEL performance can be enhanced further by employing external focussing stronger than that given by Eq. (4).

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Table 1. SASE Amplifier Parameters*

λ (Å)	1000	100	50	20	10
E (GeV)	0.325	0.8	1.1	1.8	3.25
P_u (MW)	450	360	300	175	0.65
L_{sat}(m)	4.4	11.6	16.7	28.4	47.0
L_{tap} (m)	5.2	18.4	13.3	0	0
P_{tap} (GW)	30	36	9.3	0	0
L_{gain}	0.24	0.66	1.0	1.8	3.1
λ_u (cm)	2.0	1.75	1.75	1.75	2.0
L_{tot} (m)	9.6	30	30	30	30

*electron beam parameter $\gamma\varepsilon_x$ = 1.5 mm-mrad, I = 1000 A, σ_γ/γ = 2.2 × 10^{-4}.
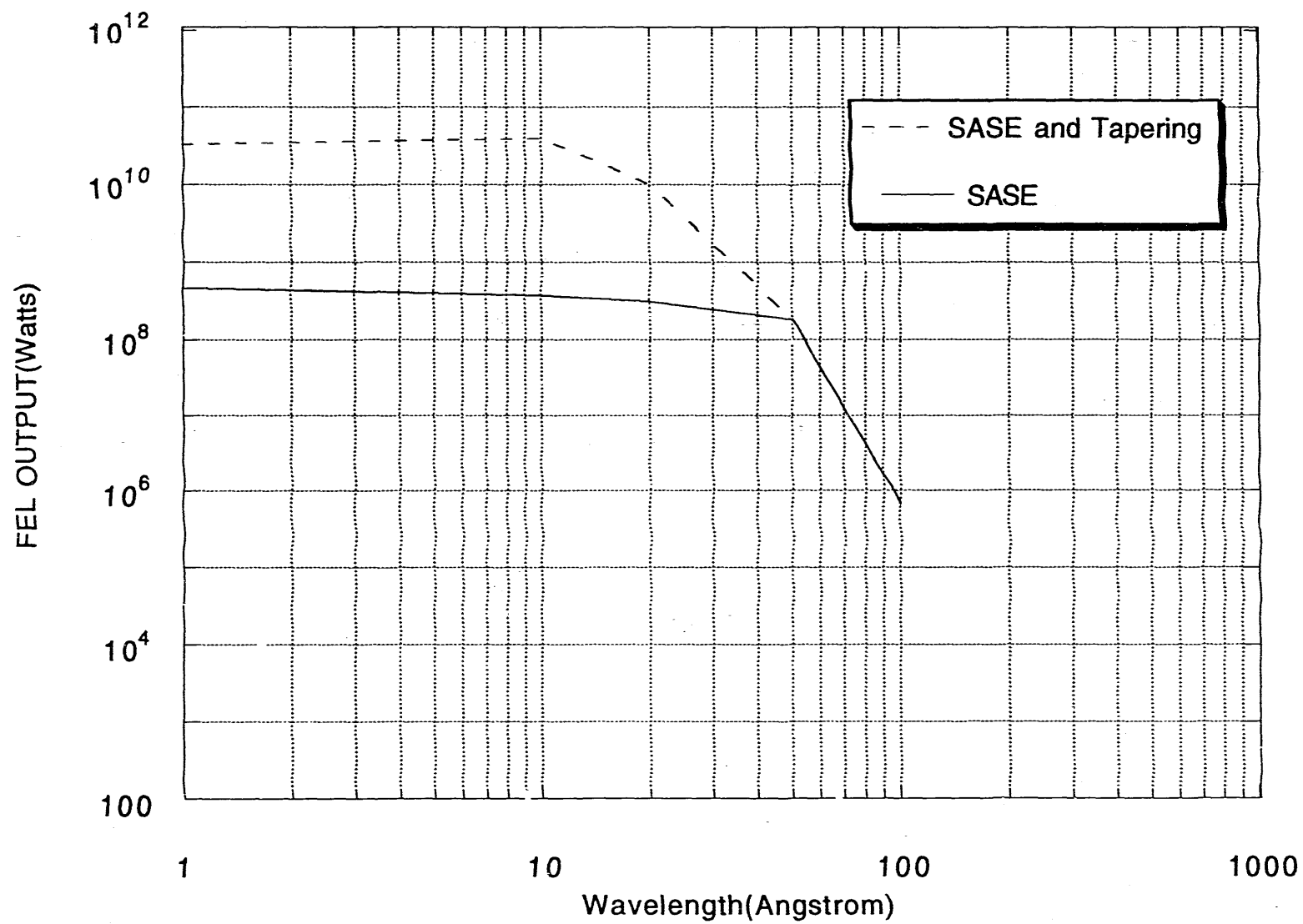
FEL OUTPUT

Figure 1